

Tugas Pertemuan 1

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Persamaan Diferensial Biasa (c)

Selesaikan : $\frac{dy}{dx} = \frac{y + \sqrt{x^2 - y^2}}{x}$

$$\rightarrow \frac{dy}{dx} = \frac{y + \sqrt{x^2 - y^2}}{x}$$

$$\frac{dy}{dx} = \frac{y}{x} + \frac{\sqrt{x^2 - y^2}}{x}$$

$$\frac{dy}{dx} = \frac{y}{x} + \sqrt{\frac{x^2}{x^2} - \frac{y^2}{x^2}}$$
$$= \frac{y}{x} + \sqrt{1 - \left(\frac{y}{x}\right)^2}$$

* $y = vx \rightarrow v = \frac{y}{x}$

$$\frac{dy}{dx} = \frac{dv}{dx} \cdot x + v$$

$$\frac{dv}{dx} x + v = v + \sqrt{1 - v^2}$$

$$\frac{1}{\sqrt{1 - v^2}} dv = \frac{1}{x} dx$$

$$\sin^{-1} v = \ln x + C$$

$$C = \sin^{-1} v - \ln x$$

$$C = \sin^{-1}\left(\frac{y}{x}\right) - \ln(x)$$

$$\ln C + \ln x = \sin^{-1}\left(\frac{y}{x}\right)$$

$$\ln(Cx) = \sin^{-1}\left(\frac{y}{x}\right)$$

$$\ln(Cx) = \arcsin\left(\frac{y}{x}\right)$$

• Pembuktian persamaan homogen :

$$\frac{dy}{dx} = \frac{y + \sqrt{x^2 - y^2}}{x}$$

$$x dy = \underbrace{y + \sqrt{x^2 - y^2}}_{N(x, y)} dx$$
$$\underbrace{x dy}_{M(x, y)}$$

$$M(\lambda x, \lambda y) = \lambda x$$

$$N(\lambda x, \lambda y) = \lambda y + \sqrt{\lambda^2 x^2 - \lambda^2 y^2}$$
$$= \lambda y + \lambda \sqrt{x^2 - y^2}$$
$$= \lambda [y + \sqrt{x^2 - y^2}]$$

Terbukti

$$\text{PUDP : } \arcsin\left(\frac{y}{x}\right) = \ln(Cx)$$